Pair creation of higher dimensional black holes on a de Sitter background

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We study in detail the quantum process in which a pair of black holes is created in a higher D-dimensional de Sitter (dS) background. The energy to materialize and accelerate the pair comes from the positive cosmological constant. The instantons that describe the process are obtained from the Tangherlini black hole solutions. Our pair creation rates reduce to the pair creation rate for Reissner-Nordström−dS solutions when $D = 4$. Pair creation of black holes in the dS background becomes less suppressed when the dimension of the spacetime increases. The dS space is the only background in which we can discuss analytically the pair creation process of higher dimensional black holes, since the C-metric and the Ernst solutions, which describe, respectively, a pair accelerated by a string and by an electromagnetic field, are not known yet in a higher dimensional spacetime.

I. INTRODUCTION

String theories have boosted the interest on higher dimensional spacetimes. This interest was renewed recently in connection to the TeV-scale theory [1] which suggests that the universe in which we live may have large extra dimensions. According to this conjecture, we would live on a four-dimensional submanifold, where the standard model inhabits, whereas the gravitational degrees of freedom propagate throughout all dimensions. Studies on higher dimensional spacetimes have also raised the possibility that future accelerators, such as the Large Hadron Collider (LHC) at CERN produce black holes, and thus detect indirectly gravitational waves [2].

Higher dimensional black holes, in particular, are providing a useful background to test the possible existence of extra dimensions. It is therefore important to discuss processes that might lead to the production of black holes in a higher dimensional background. Among these there is one that deserves special attention: the quantum Schwinger-like process of black hole pair creation in an external field. This process allows the production of black holes with any mass, including masses that are well below the Chandrasekhar limiting mass. In order to turn a pair of virtual black holes into a real pair, one needs a background field that provides the energy needed to materialize the pair, and that furnishes the force necessary to accelerate away the black holes once they are created. In a four-dimensional spacetime, the pair creation process has been discussed in several background fields, which can

be: (i) an external electromagnetic field with its Lorentz force (see [3−13]), (ii) the positive cosmological constant Λ, or inflation (see [14−22]), (iii) a cosmic string with its tension (see [23−30]), (iv) a domain wall with its gravitational repulsive energy (see [31−34]). One can also have a combination of the above fields, for example, a process involving cosmic string breaking in a background magnetic field [35], or a scenario in which a cosmic string breaks in a cosmological background [36,37].

To study the pair creation process, one needs solutions that describe appropriately the evolution of the black hole pair after its creation, i.e., one needs solutions that represent a pair of black holes accelerated by the background field responsible for the pair production. These solutions include the C-metric [23] and the Ernst solution [3], which describe a pair accelerated by a string and by an electromagnetic field, respectively. Unfortunately, the higher dimensional C-metric and Ernst solutions have not been found yet. Therefore, pair creation of higher dimensional black holes in a cosmic string background or in an electromagnetic background cannot be discussed analytically. However, the cosmological expansion provided by a de Sitter (dS) background spacetime can also furnish the energy necessary for the pair creation process. In this case, the solution that describes the black hole pair after its creation is well known, it is the Reissner-Nordström−de Sitter solution in higher dimensions [38], and can be used to study the pair creation process of higher dimensional black holes in a dS background. This task will be carried in this paper. In $D = 4$ (D is the dimension of the spacetime), the pair creation rates have been previously computed in [17].

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Pair creation of black holes is also a good source of gravitational radiation emission. In a higher dimensional flat background, an estimate for the amount of gravitational radiation released during the pair creation period has been given in [39]. After the pair creation, the black hole pair accelerates away and, consequently, the black holes continue to release energy. In a four-dimensional dS background, the gravitational radiation emitted by uniformly accelerated black holes has been computed in [40].

The plan of this paper is as follows. In Sec. II we briefly review some properties of the higher dimensional dS black holes that are used in the paper. In Sec. III we construct the instantons that describe the pair creation process. In Sec. IV, we explicitly evaluate the pair creation rate of black holes. In Sec. VI concluding remarks are presented. Throughout this paper we use units in which the D-dimensional Newton's constant is equal to one, and $c = 1$.

II. HIGHER DIMENSIONAL dS BLACK HOLES

In an asymptotically de Sitter background (positive cosmological constant, $\Lambda > 0$), the most general static higher dimensional black hole solution with spherical topology was found by Tangherlini [38]. The gravitational field is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{D-2}^2, \quad (1)$$

where D is the dimension of the spacetime, $d\Omega_{D-2}^2$ is the line element on a unit $(D - 2)$ sphere, $d\Omega_{D-2}^2 = d\theta_1^2 + \cdots + \prod_{i=1}^{D-3} \sin^2\theta_i d\theta_{D-2}^2$, and the function $f(r)$ is given by

$$f(r) = 1 - \frac{\Lambda}{3}r^2 - \frac{M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}. \quad (2)$$

The mass parameter M and the charge parameter Q are related to the ADM mass, M_{ADM}, and ADM electric charge, Q_{ADM}, of the solution by [41]

$$M_{ADM} = \frac{(D-2)\Omega_{D-2}}{16\pi}M,$$
$$Q_{ADM} = \sqrt{\frac{(D-3)(D-2)}{2}}Q, \quad (3)$$

where Ω_{D-2} is the area of a unit $(D - 2)$ sphere,

$$\Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \quad (4)$$

and $\Gamma[z]$ is the gamma function. The radial electromagnetic field produced by the electric charge Q_{ADM} is given by

$$F = -\frac{Q_{ADM}}{r^{D-2}}dt \wedge dr. \quad (5)$$

These solutions have a curvature singularity at the origin, and the black hole solutions can have at most three horizons, the Cauchy horizon r_-, the event horizon r_+, and the cosmological horizon r_c, that satisfy $r_- \leq r_+ \leq r_c$.

Of particular interest for us are the D-dimensional extreme dS-Tangherlini black holes, for which at least two of the horizons coincide. If we label this degenerate horizon by ρ, the extreme black holes can be identified by describing the parameters M and Q as a function of ρ [42]. More concretely, the extreme black holes satisfy the relations [42]

$$M = 2\rho^{D-3}\left(1 - \frac{D-2}{D-3}\frac{\Lambda}{3}\rho^2\right),$$
$$Q^2 = \rho^{2(D-3)}\left(1 - \frac{D-1}{D-3}\frac{\Lambda}{3}\rho^2\right), \quad (6)$$

where the condition $Q^2 \geq 0$ implies that $\rho \leq \rho_{max}$ with

$$\rho_{max} = \sqrt{\frac{D-3}{D-1}\frac{3}{\Lambda}}. \quad (7)$$

Since a dS black hole can have at most three horizons, one can have three distinct extreme dS black holes, namely, the cold black hole with $r_- = r_+ \equiv \rho$, the ultracold black hole in which the three horizons coincide, $r_- = r_+ = r_c \equiv \rho$, and the Nariai black hole with $r_+ = r_c \equiv \rho$ (here we follow the nomenclature used in the analogous four-dimensional black holes [16]). For the D-dimensional cold black hole ($r_- = r_+$), M and Q increase with ρ, and one has [42]

$$0 < \rho < \rho_u, \qquad 0 < M < \frac{4}{D-1}\rho_u^{D-3},$$
$$0 < Q < \frac{1}{\sqrt{D-2}}\rho_u^{D-3}, \quad (8)$$

where we have defined

$$\rho_u = \sqrt{\frac{3}{\Lambda}}\frac{D-3}{\sqrt{(D-2)(D-1)}}. \quad (9)$$

For the D-dimensional ultracold black hole ($r_- = r_+ = r_c$), one has [42]

$$\rho = \rho_u, \qquad M = \frac{4}{D-1}\rho_u^{D-3}, \qquad Q = \frac{1}{\sqrt{D-2}}\rho_u^{D-3}. \quad (10)$$

Finally, for the D-dimensional Nariai black hole ($r_+ = r_c$), M and Q decrease with ρ, and one has [42]

$$\rho_u < \rho \leq \rho_{max}, \qquad \frac{2}{D-1}\rho_{max}^{D-3} \leq M < \frac{4}{D-1}\rho_u^{D-3},$$
$$0 \leq Q < \frac{1}{\sqrt{D-2}}\rho_u^{D-3}. \quad (11)$$

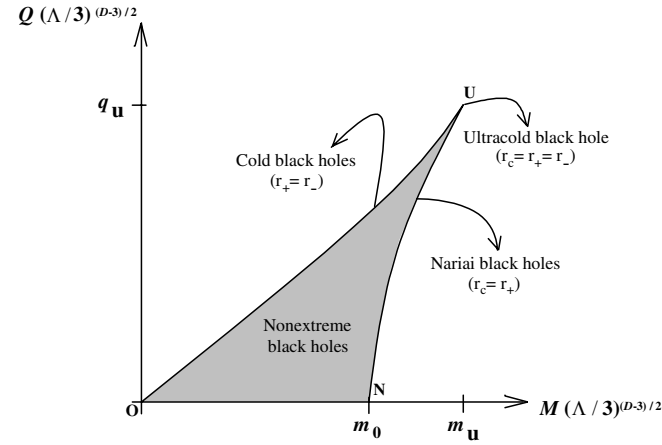The ranges of M and Q that represent each one of the above extreme black holes are sketched in Fig. 1.

FIG. 1.　Range of M and Q for which one has a nonextreme black hole (region interior to the closed line $ONUO$), an extreme Nariai black hole with $r_+ = r_c$ (line NU), an extreme cold black hole with $r_- = r_+$ (line OU), and an extreme ultracold black hole with $r_- = r_+ = r_c$ (point U). The line ON represents the nonextreme dS-Schwarzschild black hole, and point N represents the extreme Nariai Schwarzschild black hole. The nonshaded area represents a naked singularity region. The constants in the axes are $m_0 = \frac{2}{D-1}(\frac{D-3}{D-1})^{(D-3)/2}$, $m_u = \frac{4}{D-1} \times (\frac{(D-3)^2}{(D-2)(D-1)})^{(D-3)/2}$, and $q_u = (1/\sqrt{D-2})(\frac{(D-3)^2}{(D-2)(D-1)})^{(D-3)/2}$.

III. HIGHER DIMENSIONAL dS INSTANTONS

In order to evaluate the black hole pair creation rate, we need to find the instantons of the theory, i.e., we must look into the Euclidean section of the higher dimensional dS solution and choose only those Euclidean solutions which are regular in a way that will be explained soon. To obtain the Euclidean section of the dS solution from the Lorentzian dS solution, (1), (2), and (5), we simply introduce an imaginary time coordinate $\tau = -it$. To have a positive definite Euclidean metric we must, in general, require that r belongs to $r_+ \leq r \leq r_c$.

A regular instanton solution is obtained when we eliminate the conical singularities that appear in the horizons. This is achieved by choosing appropriately the period of τ. For example, to eliminate the conical singularity at $r = r_c$ the period of τ must be $\beta = 2\pi/k_c$, where k_c is the surface gravity of the cosmological horizon, i.e.,

$$\beta = \frac{4\pi}{|f'(r_c)|}, \qquad (12)$$

and, in general, this period also eliminates the conical singularity at the horizon r_+ if $|f'(r_+)| = |f'(r_c)|$.

We can construct four distinct regular instantons. Below, we will describe each one of these four instantons, following the order: (A) cold instanton, (B) Nariai instanton, (C) ultracold instanton, and (D) lukewarm instanton. These instantons are the higher dimensional counterparts of the four-dimensional instantons that have been constructed [15–17,21] from the Euclidean section of the

dS–Reissner-Nordström solution, and thus we preserve the four-dimensional nomenclature. The cold instanton, the Nariai instanton, and the ultracold instanton are obtained by Euclidean-izing solutions found in [42]. These four families of instantons will allow us to calculate the pair creation rate of accelerated dS–Reissner-Nordström black holes in Sec. IV.

A. The higher dimensional cold instanton

We start with the case $r_- = r_+$ and $r_c \neq r_+$. This solution is the cold instanton, and requires the presence of charge. The gravitational field of the higher dimensional cold instanton is given by (1) and (2), while its Maxwell field is given by (5), with the replacement $\tau = -it$. Moreover, the degenerated horizon ρ, the mass parameter M, and the charge parameter Q satisfy relations (6) and (8).

In the cold instanton, the allowed range of r in the Euclidean sector is simply $r_+ < r \leq r_c$. This occurs because, when $r_- = r_+$, the proper distance along spatial directions between r_+ and r_c goes to infinity. The point $r = r_+ = \rho$ disappears from the τ, r section which is no longer compact. Thus, in this case we have a conical singularity only at r_c, and so we obtain a regular Euclidean solution by simply requiring that the period of τ is equal to (12). The topology of the higher dimensional cold instanton is $\mathbb{R}^2 \times S^{D-2}$ ($0 \leq \tau \leq \beta$, $r_+ < r \leq r_c$). The surface $r = r_+ = \rho$ is then an internal infinity boundary that will have to be taken into account in the calculation of the action of the cold instanton (see Sec. IVA). The Lorentzian sector of this cold case describes two extreme ($r_- = r_+$) dS black holes being accelerated by the cosmological background, and the higher dimensional cold instanton describes pair creation of these extreme black holes. To compute the pair creation rate of cold black holes, we need to know the location of the cosmological horizon, r_c. This location can be explicitly determined for $D = 4$ and $D = 5$. Specifically, for $D = 4$ one has $r_c = \sqrt{3/\Lambda - 2\rho^2} - \rho$, where, from (8), one has $0 < \rho < 1/\sqrt{2\Lambda}$. For $D = 5$, one has $r_c = \sqrt{3/\Lambda - 2\rho^2}$, where, from (8), one has $0 < \rho < 1/\sqrt{\Lambda}$. For $D \geq 6$ finding explicitly r_c requires solving a polynomial of degree higher than four.

B. The higher dimensional Nariai instanton

We now turn our attention to the case $r_+ = r_c$ and $r_- \neq r_+$. This solution is called Nariai instanton, and it exists with or without charge. When the charge vanishes the only regular Euclidean solution that can be constructed is the neutral Nariai instanton. As we said in the beginning of this section, one requires that $r_+ \leq r \leq r_c$ in order to obtain a positive definite metric. But in the Nariai case $r_+ = r_c$, and so it seems that we are left with no space to work with in the Euclidean sector. However, it can be shown that the proper distance between r_+ and r_c remains finite as $r_+ \rightarrow r_c$ [12,14,17,42].

The gravitational field of the higher dimensional Nariai instanton is given by [42]

$$ds^2 = \frac{1}{A}(\sin^2\chi d\tau^2 + d\chi^2) + \frac{1}{B}d\Omega_{D-2}^2, \quad (13)$$

where χ runs from 0 to π, and A and B are related to Λ and Q by [42]

$$\Lambda = \frac{3}{(D-2)(D-1)}[A + (D-3)^2 B],$$

$$Q^2 = \frac{(D-3)B - A}{(D-3)(D-2)B^{D-2}}. \quad (14)$$

One has $B = \rho^{-2}$, where ρ lies in the range defined in (11) [42]. In particular, the neutral Nariai solution satisfies $B = \rho_{\max}^2$, with $\rho_{\max}$ defined in (7), and $A = (D-1)\Lambda/3$. The Maxwell field (5) of the higher dimensional charged Nariai solution is

$$F = iQ_{\mathrm{ADM}}\frac{B^{(D-2)/2}}{A}\sin\chi d\tau \wedge d\chi. \quad (15)$$

So, if we give the parameters Λ and Q, we can construct the higher dimensional Nariai solution. The topology of the Nariai instanton is $S^2 \times S^{D-2}$ ($0 \leq \tau \leq \beta$, $0 \leq \chi \leq \pi$). The Lorentzian sector of this solution is the direct topological product of $dS_2 \times S^{D-2}$, i.e., of a $(1 + 1)$-dimensional de Sitter spacetime with a $(D - 2)$ sphere of fixed size. To each point in the sphere corresponds a dS_2 spacetime. In the $D = 4$ case, it has been shown [14,18] that the Nariai solution decays through the quantum tunneling process into a slightly nonextreme dS black hole pair (for a complete review on this subject, see [43]). We then naturally expect that an analogous quantum instability is present in the higher dimensional Nariai solution, as will be shown. Therefore, the Nariai instanton describes the creation of a higher dimensional Nariai universe that then decays into a slightly nonextreme ($r_+ \sim r_c$) pair of black holes accelerated by the cosmological constant background.

C. The higher dimensional ultracold instanton

The third regular instanton is defined by (C): $r_- = r_+ = r_c$. It is regular when condition (12) is satisfied. This is the ultracold instanton, and can be viewed as a limiting case of both the cold instanton and the charged Nariai instanton.

The gravitational field of the higher dimensional ultracold instanton is given by [42]

$$ds^2 = \chi^2 d\tau^2 + d\chi^2 + \rho_u^2 d\Omega_{D-2}^2, \quad (16)$$

where χ runs from 0 to $+\infty$, and ρ_u is defined in (9). The Maxwell field of the higher dimensional ultracold instanton is

$$F = i\frac{Q_{\mathrm{ADM}}}{\rho_u^{D-2}}\chi d\tau \wedge d\chi, \quad (17)$$

where Q_{ADM} is given by (3) and (10). So, if we give Λ we can construct the higher dimensional ultracold instanton. Notice that the spacetime factor $\chi^2 dT^2 + d\chi^2$ is just $\mathbb{E}^2$ (two-dimensional Euclidean space) in Rindler coordinates. Therefore, under the usual coordinate transformation $\chi = \sqrt{x^2 - t^2}$ and $\tau = \mathrm{arctanh}(t/x)$, this factor transforms into $dt^2 + dx^2$. The topology of the ultracold instanton is $\mathbb{E}^2 \times S^{D-2}$. The Lorentzian sector of this solution is the direct topological product of $\mathbb{M}^{(1,1)} \times S^{D-2}$, i.e., of a $(1 + 1)$-dimensional Minkowski spacetime with a $(D - 2)$ sphere of fixed size. The ultracold instanton describes the creation of a higher dimensional Nariai–Bertotti-Robinson universe [42] that then decays into a slightly nonextreme ($r_- \sim r_+ \sim r_c$) pair of black holes accelerated by the cosmological constant background.

D. The higher dimensional lukewarm instanton

Finally, we have the lukewarm instanton. This instanton satisfies $r_- \neq r_+ \neq r_c$, and

$$f'(r_+) = -f'(r_c). \quad (18)$$

In this case the surface gravities of the horizons r_+ and r_c are equal ($k_+ = k_c$), and thus they have equal Hawking temperature. The choice (12) for the period of τ also eliminates the conical singularity at the outer black hole horizon, $r = r_+$. The solution is then regular in the whole Euclidean range $r_+ \leq r \leq r_c$. The topology of the lukewarm instanton is $S^2 \times S^{D-2}$ ($0 \leq \tau \leq \beta$, $r_+ \leq r \leq r_c$). The Lorentzian sector of the lukewarm solution describes two higher dimensional dS black holes being accelerated apart by the cosmological constant, so this instanton describes pair creation of nonextreme black holes.

The gravitational field of the higher dimensional lukewarm instanton is given by (1) with the requirement that $f(r)$ satisfies condition (18) and $f(r_+) = 0 = f(r_c)$. To find the properties of the lukewarm instanton we note that the function $f(r)$, given by (1), can also be written as

$$f(r) = -\frac{\Lambda}{3}r^2\left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_c}{r}\right)\left(1 + \frac{a_1}{r} + \frac{a_2}{r^2}\right.$$

$$\left. + \cdots + \frac{a_{2(D-3)}}{r^{2(D-3)}}\right), \quad (19)$$

where a_i [$i = 1, \ldots, 2(D - 3)$] are constants that can be found from the matching between (1) and (19). This matching, together with the extra condition (18), leads to unique relations between the parameters Λ, M, Q and the position of the horizons, r_+ and r_c. Since this procedure involves polynomials with a high degree, we have not been able to find the general relations between (Λ, M, Q) and (r_+, r_c) for any D. So, we have to carry this procedure for each D. As examples, we specifically discuss now the $D = 4$ and the $D = 5$ lukewarm instantons. For $D = 4$, the above procedure yields the relations

$$\Lambda = \frac{3}{(r_c + r_+)^2}, \qquad M = 2\frac{r_c r_+}{r_c + r_+}, \qquad Q = \frac{r_c r_+}{r_c + r_+}.$$
(20)

For $D = 5$, the relations are

$$\Lambda = 3\left(2r_c + r_+ - \frac{r_c^3(r_c + r_+)}{r_c^2 + r_c r_+ + r_+^2}\right)^{-1},$$

$$M = \frac{r_c^2 r_+^2 (2r_c^2 + r_c r_+ + 2r_+^2)}{r_c^4 + r_c^3 r_+ + 3r_c^2 r_+^2 + r_c r_+^3 + r_+^4},$$
(21)

$$Q^2 = \frac{r_c^4 r_+^4}{r_c^4 + r_c^3 r_+ + 3r_c^2 r_+^2 + r_c r_+^3 + r_+^4}.$$

These two examples indicate an important difference between the lukewarm instanton in $D = 4$ dimensions and in $D \geq 5$: for $D = 4$ the lukewarm instanton has a ADM mass, $M_{ADM} = M/2$, equal to its ADM charge, $Q_{ADM} = Q$, while for $D \geq 5$ one has $M_{ADM} \neq Q_{ADM}$. Note also that relations (20) and (21) and their higher dimensional counterparts define implicitly r_c and r_+ as a function of Λ, M, and Q. The location of r_c and r_+ can be explicitly determined for $D = 4$ and $D = 5$. For $D \geq 6$ finding explicitly r_c requires solving a polynomial of degree higher than four.

IV. CALCULATION OF THE BLACK HOLE PAIR CREATION RATES

The pair creation rate of higher dimensional black holes in a dS background is given, according to the no-boundary proposal of [44], by

$$\Gamma = \eta e^{-2I_{inst} + 2I_{dS}},$$
(22)

where η is the one-loop contribution from the quantum quadratic fluctuations in the fields that will not be considered here (for the computation of this factor in some special cases, see [14,20,45]). I_{inst} is the classical Euclidean action of the gravitational instanton that mediates the pair creation of black holes, given by [10,12]

$$I_{inst} = -\frac{1}{16\pi} \int_{\mathcal{M}} d^D x \sqrt{g}(R - 2\lambda - F^{\mu\nu}F_{\mu\nu}) - \frac{1}{8\pi}$$

$$\times \int_{\Sigma = \partial\mathcal{M}} d^{D-1}x\sqrt{h}K - \frac{1}{4\pi}$$

$$\times \int_{\Sigma = \partial\mathcal{M}} d^{D-1}x\sqrt{h}F^{\mu\nu}n_\mu A_\nu,$$
(23)

where $\Sigma = \partial\mathcal{M}$ is the boundary of a compact manifold $\mathcal{M}$, g is the determinant of the Euclidean metric, h is the determinant of the induced metric on the boundary Σ, R is the Ricci scalar, K is the trace of the extrinsic curvature K_{ij} of the boundary, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell field strength of the gauge field A_ν, n_μ is the unit outward normal to Σ, and we have defined

$$\lambda = \frac{(D-1)(D-2)\Lambda}{6}.$$
(24)

The coefficient of the Λ term was chosen in order to ensure that, for any dimension D, the pure dS spacetime is described by $f(r) = 1 - (\Lambda/3)r^2$, as occurs with $D = 4$. The expression for the Ricci scalar in D dimensions is worthy of a comment. Variation of the action (23) yields the equation for the gravitational field, $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci tensor and $T_{\mu\nu}$ is the electromagnetic energy-momentum tensor, $T_{\mu\nu} = \frac{1}{4\pi}(g^{\alpha\beta}F_{\alpha\mu}F_{\beta\nu} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta})$. The contraction of the Einstein equation with $g^{\mu\nu}$ yields the Ricci tensor

$$R = \frac{D(D-1)}{3}\Lambda - \frac{16\pi}{D-2}T,$$
(25)

where T is the trace of $T_{\mu\nu}$. We note that in a general D-dimensional background the electromagnetic energy-momentum tensor is not traceless. Indeed, the contraction of $T_{\mu\nu}$ with $g^{\mu\nu}$ yields

$$T = -\frac{D-4}{4\pi}F_{\mu\nu}F^{\mu\nu},$$
(26)

which vanishes only for $D = 4$.

In (22), I_{dS} is the Euclidean action of the S^D gravitational instanton that mediates the nucleation of a dS space from nothing. Use of (2), (12), and (25), with $M = 0$ and $Q = 0$, yields for I_{dS} the value

$$I_{dS} = -\frac{1}{16\pi} \int d^D x \sqrt{g}(R - 2\lambda)$$

$$= -\frac{1}{4}\left(\frac{3}{\Lambda}\right)^{(D-2)/2} \frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}.$$
(27)

A. The higher dimensional cold pair creation rate

The Maxwell field of the higher dimensional cold is $F = -i(Q_{ADM}/r^{D-2})d\tau \wedge dr$. With this information we are able to compute all of the terms of the Euclidean action (23). We start with

$$-\frac{1}{16\pi} \int_{\mathcal{M}} d^D x \sqrt{g}(R - 2\lambda)$$

$$= \left(-\frac{D-1}{24\pi}\Lambda + \frac{(D-4)(D-3)}{16\pi}Q^2\right) \int d\Omega_{D-2}$$

$$\times \int_0^{\beta/2} d\tau \int_\rho^{r_c} dr\, r^{D-2}$$

$$= \frac{\pi^{(D-3)/2}}{\Gamma[(D-1)/2]} \frac{\beta}{8}\left[-\frac{\Lambda}{3}(r_c^{D-1} - \rho^{D-1})\right.$$

$$\left. + \frac{(D-4)Q^2}{2}(\rho^{-(D-3)} - r_c^{-(D-3)})\right],$$
(28)

where $\int d\Omega_{D-2} = \Omega_{D-2}$ is defined in (4). The Maxwell term in the action yields

$$\frac{1}{16\pi} \int_{\mathcal{M}} d^D x \sqrt{g} F^2 = -\frac{(D-2)Q^2\beta}{16} \frac{\pi^{(D-3)/2}}{\Gamma[(D-1)/2]}$$
$$\times [\rho^{-(D-3)} - r_{\mathrm{c}}^{-(D-3)}], \qquad (29)$$

and $\int_\Sigma d^{D-1}x\sqrt{h}K = 0$. In order to compute the extra Maxwell boundary term in (23), we have to find a vector potential, A_ν, that is regular everywhere including at the horizons. An appropriate choice in the cold case is $A_r = -i(Q_{\mathrm{ADM}}/r^{D-2})\tau$. The integral over Σ consists of an integration between ρ and r_{c} along the $\tau = 0$ surface and back along $\tau = \beta/2$, and of an integration between $\tau = 0$ and $\tau = \beta/2$ along the $r = r_{\mathrm{c}}$ surface and back along the $r = \rho$ surface. The normal to Σ_τ is $n_\mu = (\sqrt{f(r)}, 0, \ldots, 0)$, and the normal to Σ_h is $n_\mu = (0, \sqrt{f(r)}, 0, \ldots, 0)$. Thus, the nonvanishing contribution comes only from the integration along the $\tau = \beta/2$ surface. The Maxwell boundary term in (23) is then

$$-\frac{1}{4\pi} \int_{\Sigma_{\tau=\beta/2}} d^{D-1}x\sqrt{h}F^{\tau r}n_\tau A_r = -\frac{1}{8\pi} \int_{\mathcal{M}} d^D x\sqrt{g} F^2.$$
$$(30)$$

Adding all of these terms yields the action (23) of the higher dimensional cold instanton (onwards the subscript "c" means cold)

$$I_{\mathrm{c}} = -\frac{r_{\mathrm{c}}^{D-2}}{4} \frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \qquad (31)$$

which, for $D = 4$, reduces to the result of [17]. The allowed interval of ρ is defined in (8). As ρ varies from $\rho = \rho_{\mathrm{u}}$, defined in (9), to $\rho = 0$, the cold action (31) varies according to

$$-\frac{\rho_{\mathrm{u}}^{D-2}}{4} \frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]} < I_{\mathrm{c}} < I_{\mathrm{dS}}, \qquad (32)$$

where the lower limit of this relation is the ultracold action, as we shall see in (44), and I_{dS} is defined in (27).

The pair creation rate of extreme cold black holes is given by (22),

$$\Gamma_{\mathrm{c}} = \eta e^{-2I_{\mathrm{c}} + 2I_{\mathrm{dS}}}, \qquad (33)$$

where η is the one-loop contribution not computed here.

B. The higher dimensional Nariai pair creation rate

The first term of the Euclidean action (23) gives in the Nariai case

$$-\frac{1}{16\pi} \int_{\mathcal{M}} d^D x\sqrt{g}(R - 2\lambda)$$
$$= \left(-\frac{\Lambda(D-1)}{24\pi} + \frac{(D-4)(D-3)}{16\pi} Q^2 B^{D-2} \right)$$
$$\times \int d\Omega_{D-2} \int_0^{2\pi/2} d\tau \int_0^\pi d\chi \frac{\sin\chi}{AB^{(D-2)/2}}$$
$$= \frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]} \left[-\frac{\Lambda(D-1)}{6} \frac{1}{AB^{(D-2)/2}} \right.$$
$$\left. + \frac{Q^2(D-4)(D-3)}{4} \frac{B^{(D-2)/2}}{A} \right]. \qquad (34)$$

The Maxwell term in the action yields

$$\frac{1}{16\pi} \int_{\mathcal{M}} d^D x\sqrt{g} F^2 = -\frac{(D-2)(D-3)Q^2}{4} \frac{B^{(D-2)/2}}{A}$$
$$\times \frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \qquad (35)$$

and $\int_\Sigma d^{D-1}x\sqrt{h}K = 0$. In order to compute the extra Maxwell boundary term in (23), we have to find a vector potential, A_ν, that is regular everywhere including at the horizons. An appropriate choice in the Nariai case is $A_\chi = iQ_{\mathrm{ADM}}(B^{(D-2)/2}/A)\sin\chi\tau$. The integral over Σ consists of an integration between $\chi = 0$ and $\chi = \pi$ along the $\tau = 0$ surface and back along $\tau = \pi$, and of an integration between $\tau = 0$ and $\tau = \pi$ along the $\chi = 0$ surface, and back along the $\chi = \pi$ surface. The nonvanishing contribution to the Maxwell boundary term in (23), $-\frac{1}{4\pi} \times \int_\Sigma d^3x\sqrt{h}F^{\mu\nu}n_\mu A_\nu$, comes only from the integration along the $\tau = \pi$ surface and is given by

$$-\frac{1}{4\pi} \int_{\Sigma_{\tau=\pi}} d^{D-1}x\sqrt{h}F^{\tau\chi}n_\tau A_\chi = -\frac{1}{8\pi} \int_{\mathcal{M}} d^D x\sqrt{g} F^2.$$
$$(36)$$

Adding all of these terms yields the action (23) of the higher dimensional Nariai instanton (onwards the subscript "N" means Nariai)

$$I_{\mathrm{N}} = -\frac{1}{2B^{(D-2)/2}} \frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \qquad (37)$$

which, for $D = 4$, reduces to the result of [12,17]. One has $B = \rho^{-2}$, where ρ lies in the range defined in (11). Thus, the Nariai action (37) lies in the range

$$-\frac{\pi^{(D-1)/2}\rho_{\max}^{D-2}}{2\Gamma[(D-1)/2]} \le I_{\mathrm{N}} < -\frac{\pi^{(D-1)/2}\rho_{\mathrm{u}}^{D-2}}{2\Gamma[(D-1)/2]}, \qquad (38)$$

where the quantities $\rho_{\max}$ and ρ_{u} are defined, respectively, in (7) and (9). The equality holds in the neutral Nariai case ($Q = 0$), and this case has been previously discussed in [46], while the upper limit of (38) is twice the value of the ultracold action, which will be defined in (44).

The pair creation rate of extreme Nariai black holes is given by (22),

$$\Gamma_{\mathrm{N}} = \eta e^{-2I_{\mathrm{N}}+2I_{\mathrm{dS}}}, \tag{39}$$

where I_{dS} is given by (27), and η is the one-loop contribution not computed here. The process studied in this subsection describes the nucleation of a higher dimensional Nariai universe that is unstable [14,18,43] and decays through the pair creation of extreme Nariai black holes.

C. The higher dimensional ultracold pair creation rate

The boundary $\Sigma = \partial\mathcal{M}$ that appears in (23) consists of an initial spatial surface at $\tau = 0$ plus a final spatial surface at $\tau = \pi$. We label these two 3-surfaces by Σ_τ. Each one of these two spatial $(D-1)$-surfaces is bounded by a $(D-2)$-surface at the Rindler horizon $\chi = 0$ and by a $(D-2)$-surface at the internal infinity $\chi = \infty$. The two surfaces Σ_τ are connected by a timelike $(D-1)$-surface, Σ_h, that intersects Σ_τ at the frontier $\chi = 0$ and by a timelike $(D-1)$-surface, $\Sigma_\infty^{\mathrm{int}}$, that intersects Σ_τ at the internal infinity boundary $\chi = \infty$. Thus $\Sigma = \Sigma_\tau + \Sigma_h + \Sigma_\infty^{\mathrm{int}}$, and the region $\mathcal{M}$ within it is compact. The first term of the Euclidean action (23) yields

$$-\frac{1}{16\pi}\int_{\mathcal{M}} d^D x\sqrt{g}(R-2\lambda)$$

$$= \frac{1}{16\pi}\left[\Lambda\frac{2(D-1)}{3}\rho_{\mathrm{u}}^{D-2} + Q^2(D-4)(D-3)\right.$$

$$\left.\times\frac{1}{\rho_{\mathrm{u}}^{D-2}}\right]\int d\Omega_{D-2}\int_0^{2\pi/2} d\tau\int_0^{\chi_0\to\infty} d\chi\chi$$

$$= \left[-\frac{D-1}{24}\Lambda\rho_{\mathrm{u}}^{D-2} + \frac{(D-4)(D-3)}{16}\frac{Q^2}{\rho_{\mathrm{u}}^{D-2}}\right]$$

$$\times\frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}\chi_0^2|_{\chi_0\to\infty}. \tag{40}$$

The Maxwell term in the action yields

$$\frac{1}{16\pi}\int_{\mathcal{M}} d^D x\sqrt{g}F^2 = -\frac{(D-3)(D-2)Q^2}{16\rho_{\mathrm{u}}^{D-2}}$$

$$\times\frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}\chi_0^2|_{\chi_0\to\infty}. \tag{41}$$

Now, contrary to the other instantons, the ultracold instanton has a nonvanishing extrinsic curvature boundary term, $-\frac{1}{16\pi}\int_\Sigma d^{D-1}x\sqrt{h}K \neq 0$, due to the internal infinity boundary ($\Sigma_\infty^{\mathrm{int}}$ at $\chi = \infty$) contribution. The extrinsic curvature to $\Sigma_\infty^{\mathrm{int}}$ is $K_{\mu\nu} = h_\mu^\alpha\nabla_\alpha n_\nu$, where $n_\nu = (0,1,0,\ldots,0)$ is the unit outward normal to $\Sigma_\infty^{\mathrm{int}}$, $h_\mu^\alpha = g_\mu^\alpha - n_\mu n^\alpha = (1,0,1,\ldots,1)$ is the projection tensor onto $\Sigma_\infty^{\mathrm{int}}$, and ∇_α represents the covariant derivative with respect to $g_{\mu\nu}$. Thus, the trace of the extrinsic curvature to $\Sigma_\infty^{\mathrm{int}}$ is $K = g^{\mu\nu}K_{\mu\nu} = \frac{1}{\chi}$, and

$$-\frac{1}{8\pi}\int_\Sigma d^{D-1}x\sqrt{h}K = -\frac{\rho_{\mathrm{u}}^{D-2}}{4}\frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}. \tag{42}$$

In the ultracold case, the vector potential A_ν, which is regular everywhere including at the horizon, needed to compute the extra Maxwell boundary term in (23) is $A_\tau = -i(Q_{\mathrm{ADM}}/\rho^{D-2})\chi^2/2$. The integral over Σ consists of an integration between $\chi = 0$ and $\chi = \infty$ along the $\tau = 0$ surface and back along $\tau = \pi$, and of an integration between $\tau = 0$ and $\tau = \pi$ along the $\chi = 0$ surface, and back along the internal infinity surface $\chi = \infty$. The nonvanishing contribution to the Maxwell boundary term in (23) comes only from the integration along the internal infinity boundary $\Sigma_\infty^{\mathrm{int}}$, and is given by

$$-\frac{1}{4\pi}\int_{\Sigma_\infty^{\mathrm{int}}} d^{D-1}x\sqrt{h}F^{\chi\tau}n_\chi A_\tau = -\frac{1}{8\pi}\int_{\mathcal{M}} d^D x\sqrt{g}F^2. \tag{43}$$

Because of the fact that $\chi_0 \to \infty$, it might seem that the contribution from (40), (41), and (43) diverges. This is not, however, the case since these three terms cancel each other. The only contribution to the action (23) of the higher dimensional ultracold instanton (onwards the subscript "u" means ultracold) comes from (42) yielding

$$I_{\mathrm{u}} = -\frac{\rho_{\mathrm{u}}^{D-2}}{4}\frac{\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \tag{44}$$

which, for $D = 4$, reduces to the result of [17]. The ultracold action coincides with the minimum value of the cold action range (32), and is equal to one-half the maximum value of the Nariai action range (38).

The pair creation rate of extreme ultracold black holes is given by (22),

$$\Gamma_{\mathrm{u}} = \eta e^{-2I_{\mathrm{u}}+2I_{\mathrm{dS}}}, \tag{45}$$

where I_{dS} is given by (27), and η is the one-loop contribution not computed here. The process studied in this subsection describes the nucleation of a higher dimensional Nariai–Bertotti-Robinson universe that is unstable, and decays through the pair creation of extreme ultracold black holes.

D. The higher dimensional lukewarm pair creation rate

The evaluation of the Euclidean action of the higher dimensional lukewarm instanton follows as in the cold case as long as we replace ρ by r_+. Therefore, the action (23) of the higher dimensional lukewarm instanton (onwards the subscript "ℓ" means lukewarm) is given by

$$I_\ell = \frac{\pi^{(D-3)/2}}{\Gamma[(D-1)/2]}\frac{\beta}{4}\left[-\frac{\Lambda(r_{\mathrm{c}}^{D-1} - r_+^{D-1})}{(D-2)(D-1)}\right.$$

$$\left.+\frac{(D-3)Q^2}{2}(r_+^{-(D-3)} - r_{\mathrm{c}}^{-(D-3)})\right]. \tag{46}$$

and the pair creation rate of nonextreme lukewarm black holes is given by (22).

E. Pair creation rate of higher dimensional nonextreme submaximal black holes

The cold, Nariai, ultracold, and lukewarm instantons are saddle point solutions free of conical singularities both in the r_+ and r_c horizons. The corresponding black holes may then nucleate in the dS background, and we have computed their pair creation rates in the last four subsections. However, these particular black holes are not the only ones that can be pair created. Indeed, it has been shown in [47] that Euclidean solutions with conical singularities may also be used as saddle points for the pair creation process. In this way, pair creation of nonextreme submaximal black holes is allowed (by this nomenclature we mean all the nonextreme black holes other than the lukewarm ones that are in the region interior to the close line $ONUO$ in Fig. 1), and their pair creation rate may be computed. In order to calculate this rate, the action is given by (23) and, in addition, it has now an extra contribution from the conical singularity (c.s.) that is present in one of the horizons (r_+, for instance) given by [14,48]

$$\frac{1}{16\pi}\int_{\mathcal{M}} d^D x \sqrt{g}(R-2\lambda)|_{\text{c.s. at } r_+} = \frac{\mathcal{A}_+\delta}{4D\pi}, \qquad (47)$$

where $\mathcal{A}_+ = \frac{2\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}r_+^{D-2}$ is the area of the $(D-2)$ sphere spanned by the conical singularity, and

$$\delta = 2\pi\left(1-\frac{\beta_c}{\beta_+}\right) \qquad (48)$$

is the deficit angle associated with the conical singularity at the horizon r_+, with $\beta_c = 4\pi/|f'(r_c)|$ and $\beta_+ = 4\pi/|f'(r_+)|$ being the periods of τ that avoid a conical singularity in the horizons r_c and r_+, respectively. The contribution from (23) follows straightforwardly in a similar way as the one shown in Sec. IV D with the period of τ, β_c, chosen in order to avoid the conical singularity at the cosmological horizon, $r = r_c$.

V. HEURISTIC DERIVATION OF THE PAIR CREATION RATES

The physical interpretation of our exact results can be clarified with a heuristic derivation of the nucleation rates. An estimate for the nucleation probability is given by the Boltzmann factor, $\Gamma \sim e^{-E_0/W_{\text{ext}}}$, where E_0 is the energy of the system that nucleates and $W_{\text{ext}} = F\ell$ is the work done by the external force F, which provides the energy for the nucleation, through the typical distance ℓ separating the created pair. We can then show that the creation probability for a black hole pair in a dS background is given by $\Gamma \sim e^{-M/\sqrt{\Lambda}}$, in agreement with the exact results. Indeed, one has $E_0 \sim 2M$, where M is the rest energy of the black hole,

and $W_{\text{ext}} \sim \sqrt{\Lambda}$ is the work provided by the cosmological background. To derive $W_{\text{ext}} \sim \sqrt{\Lambda}$, one can argue as follows. In the dS case, the Newtonian potential is $\Phi = \Lambda r^2/3$ and its derivative yields the force per unit mass or the acceleration a, Λr. This acceleration should be evaluated at the characteristic dS radius, $r = 1/\sqrt{\Lambda}$, yielding $a = \sqrt{\Lambda}$. The force can then be written as $F = \text{mass} \times \text{acceleration} \sim \sqrt{\Lambda}\sqrt{\Lambda}$, where the characteristic mass of the system is $\sqrt{\Lambda}$. Thus, the characteristic work is $W_{\text{ext}} = \text{force} \times \text{distance} \sim \Lambda(1/\sqrt{\Lambda}) \sim \sqrt{\Lambda}$, where the characteristic distance that separates the pair at the creation moment is $\sqrt{\Lambda}$. (This value follows from the fact that the dS spacetime can be represented as a hyperboloid in a Minkowski embedding spacetime, and the origin of the dS spacetime describes the hyperbolic trajectory, $X^2 - T^2 = \Lambda$, in the embedding space.) So, from the Boltzmann factor we indeed expect that the creation rate of a black hole pair in a dS background is given by $\Gamma \sim e^{-M/\sqrt{\Lambda}}$. This expression is in agreement with our results since, from (6), one has $M \sim \rho^{D-3} \sim \Lambda^{-(D-3)/2}$, and thus $\Gamma \sim e^{-M/\sqrt{\Lambda}} \sim e^{-\Lambda^{-(D-2)/2}}$.

VI. DISCUSSION OF THE RESULTS

We have studied in detail the quantum process in which a pair of black holes is created in a higher dimensional de Sitter (dS) background, a process that in $D = 4$ was previously discussed in [17]. The energy to materialize and accelerate the pair comes from the positive cosmological constant. The dS space is the only background in which we can discuss analytically the pair creation process of higher dimensional black holes, since the C-metric and the Ernst solutions, which describe, respectively, a pair accelerated by a string and by an electromagnetic field, are not known yet in a higher dimensional spacetime.

As occurs for $D = 4$, the pair creation of higher dimensional black holes is always suppressed relative to the dS space, i.e., the argument of the exponential function that defines the pair creation rate (22) is always negative. To compare the evolution of the pair creation rate as the dimension of the spacetime increases, we note that this rate goes as $\Gamma \sim e^{-\Lambda^{-(D-2)/2}}$. Thus, for a fixed value of Λ, the rate increases when D grows, i.e., pair creation of black holes in the dS background becomes less suppressed when the dimension of the spacetime increases. This behavior increases the interest of this kind of black hole creation process in a higher dimensional spacetime.

In previous works on black hole pair creation in general background fields, it has been well established that the pair creation rate is proportional to the exponential of the gravitational entropy S of the system, $\Gamma \propto e^S$, with the entropy being given by one-quarter of the total area $\mathcal{A}$ of all the horizons present in the instanton, $S = \mathcal{A}/4$. It is straightforward to verify that these relations also hold for the higher dimensional dS instantons [49]. Indeed, in the cold case, the instanton has a single horizon, the cosmo-

logical horizon at $r = r_c$, in its Euclidean section, since $r = r_+$ is an internal infinity. So, the total area of the cold instanton is $\mathcal{A}_c = \Omega_{D-2} r_c^{D-2}$. Thus, $S_c = -2I_c = \mathcal{A}_c/4$, where I_c is given by (31). In the Nariai case, the instanton has two horizons in its Euclidean section, namely, the cosmological horizon $r = r_c$ and the black hole horizon r_+, both at $r = \rho = B^{-1/2}$, and thus they have the same area. So, the total area of the Nariai instanton is $\mathcal{A}_N = 2\Omega_{D-2} B^{-(D-2)/2}$. Again, one has $S_N = -2I_N = \mathcal{A}_N/4$, where I_N is given by (37). In the ultracold case, the instanton has a single horizon in its Euclidean section, the Rindler horizon at $\chi = 0$, since $\chi = \infty$ is an internal infinity. The total area of the ultracold instanton is then $\mathcal{A}_u = \Omega_{D-2} \rho_u^{D-2}$. Thus, $S_u = -2I_u = \mathcal{A}_u/4$, where I_u is given by (44).

The ultracold instanton is a limiting case of both the charged Nariai instanton and the cold instanton (see Fig. 1). Then, as expected, the action of the cold instanton gives, in this limit, the action of the ultracold instanton [see (32)]. However, the ultracold frontier of the Nariai action is given by 2 times the ultracold action [see (38)]. The reason for this behavior is clear. Indeed, in the ultracold case and in the cold case, the respective instantons have a single horizon (the other possible horizon turns out to be a internal infinity). This horizon gives the only contribution to the total area, $\mathcal{A}$, and therefore to the pair creation rate. In the Nariai case, the instanton has two horizons with the same area, and thus the ultracold limit of the Nariai action is twice the value of the true ultracold action.

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